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One-dimensional bounce of inelastically colliding marbles on a wall

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Abstract. n marbles with zero velocity are pushed away by a wall moving at constant velocity. The collisions are treated using an inelasticity coefficient η and the equations of motion are numerically integrated until no more marbles collide with the wall. The principal investigated quantity is the relative kinetic energy E_k of the bounced marbles. A model of 'independent collision waves' is presented which predicts with a good precision the maximum of E_k for $\eta_1 = 2^{1-1/n} - 1$, and the value $\eta_2 = \tan^2 \pi/4(1-(1/n))$ at which the marbles stick to the wall. It is found that, for large n, the only important parameter is $\gamma = (1-\eta)n$. Equivalent results are found when a marble is thrown against the column.

1. Introduction

The study of materials which exhibit new and unconventional properties is of central importance in many fields of science. In this connection, there has been a rapidly growing interest in granular materials [1] and their quite unusual aspects. For instance experimental [2-5], theoretical and numerical [6, 7] researches about instabilities and segregation phenomenon [8], in vibrated particulate matter are recently witnessed. Being interested in fluidization phenomenon, our first purpose is a tentative attempt to describe a convective regime which holds inside a sandpile that becomes unstable under vertical vibrations. More precisely, we want to see if a simulation, based on inelastic collisions, can explain this experimentally observed convective motion.

Granular materials are different from simple fluids in two ways: (i) collisions between elements in the material or with the walls are inelastic; (ii) friction forces are present which, for example, are responsible of the angle of repose. We are interested in understanding the experiments of vibrated granular media and especially why and how they behave unlike simple fluids. Consequently, we study separately the two effects of inelasticity and friction. This is possible using computer simulations.

Here we want to test the first effect and see how inelastic collisions are responsible for the dynamical properties of these media. Before going to realistic three-dimensional simulation of shaked marbles, we study how a one-dimensional column of marbles bounces on a wall in absence of gravity or equivalently how this column is pushed away by a wall moving at constant velocity. Even if we relax ergodicity by reducing the dimensionality of the problem, we think that some conclusions will be qualitatively

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the same in higher dimensions. This work is the first step of a more complete study and already contains interesting features. Notice that this step can be directly used to understand what happens when a marble column bounces on a plane vibrating with a period much larger than the bounce duration. Indeed, only the relative motion of the closest marble to the plane, with respect to it, is modified by the action of gravity.

The aim of this paper is to extract the relevant parameters of the system to enhance general behaviours of the bounce and to present an analytical model which explains the main properties of the bouncing marbles. This paper is organized as follows: section 2 presents the system and the relevant parameters. Section 3 is devoted to the numerical computation results. Section 4 provides an analytical model of independent collision waves (ICW) and a way of evaluating special values of the inelasticity coefficient: (i) η_1 is the value at which the relative kinetic energy E_k of the bouncing marbles reaches its maximum; (ii) η_2 is the value at which all the marbles stick to the wall. A conclusion is drawn in the final section.

2. The inelastically colliding marbles system

The system under consideration is made up of n marbles with radius r_i and masses m_i, disposed on an x-axis in absence of external field. At time $t=0$, the position of the ith marble is x_i with $x_i > x_{i-1}$, its velocity is v_i, and the wall is at $x_w = 0$ with a velocity v_w. In the present study, the marble velocities are restricted to be identical at $t=0$. Because of the Galilean invariance, one can choose $v_w = 0$ and $v_i(t=0) = -V_0$ (with $x_i > 0$) or equivalently $v_w = V_0$ and $v_i(t=0)=0$. The separation ε_i between the marbles i and $i-1$ is defined as $\varepsilon_i = x_i - x_{i-1} - r_i - r_{i-1}$. In one dimension, the equations of motion do not depend on the marble radius, so only the separations ε_i are relevant. At $t=0$, these quantities are chosen as:

$$\varepsilon_i = \xi_i \Delta \tag{1}$$

where ξ_i is a random number in [0, 1]. A natural timescale is given by $\tau = \Delta / V_0$ and changing Δ or V_0 results in changing the timescale τ; so V_0 and Δ, in arbitrary units, have been kept fixed to 1. In the elastic case ($\eta = 1$), the bounce duration of the marble column is given by $\tau_e = \Sigma_{i=1}^{n} \varepsilon_i / V_0 = \Sigma_{i=1}^{n} \xi_i \Delta / V_0 = n\tau/2$.

The dynamics depends on the marble masses. We have investigated the case of marbles with the identical masses m_0, referred hereafter as case I, and the case of marbles with slightly distinct masses $m_i = m_0(1 + \xi_i' \delta)$, where δ ranges between 10^{-4} and 10^{-1} and ξ_i' is a random number in [−1, 1], case referred hereafter as case II.

The different collisions are characterized by the same inelasticity coefficient η defined as:

$$V_r' = -\eta V_r \tag{2}$$

where V_r and V_r' are the relative velocities between two marbles just before and just after a collision. The same coefficient is used for the collision between the wall and the first marble. The loss of kinetic energy in each collision is then $\frac{1}{2}\mu(\eta^2 - 1)V_r^2$, where μ is the reduced mass. The values of η range in the interval [0, 1] where the lower and upper limits correspond respectively to a collision without bouncing (i.e. sticking collision) and to an elastic one. The collisions are considered as instantaneous and treated via standard collision matrices $C_{i-1,i}$ between marbles $i-1$ and i; in case I one has:

$$\begin{pmatrix} v'_{i-1} \\ v'_i \end{pmatrix} = C_{i-1,i} \begin{pmatrix} v_{i-1} \\ v_i \end{pmatrix} = \begin{pmatrix} \frac{(1-\eta)}{2} & \frac{(1+\eta)}{2} \\ \frac{(1+\eta)}{2} & \frac{(1-\eta)}{2} \end{pmatrix} \begin{pmatrix} v_{i-1} \\ v_i \end{pmatrix} \qquad \text{for } i=2, n \tag{3}$$

and the collision matrix between the wall and the first marble is:

$$C_{0,1} = \begin{pmatrix} 1 & 0 \\ 1+\eta & \eta \end{pmatrix}. \tag{4}$$

For given values of n, η and initial separations $\{\varepsilon_i\}$, we let the system evolve until $v_i > v_w$, for all i, which ensures that no more collisions with the wall are possible. This final state choice is rather arbitrary and other final states are compared in the following. Then, several quantities are investigated for the bouncing column: the number N_0 of collisions between the wall and the first marble, the total number N_T of collisions, the mean final separation ε_f, the velocity V_{CM} of the centre of mass, the bounce duration τ_b, the relative kinetic energy E_k of the marbles, the first moments of the relative velocity distribution function. All these quantities are evaluated by statistical averages over initial separations (and mass distribution in case II), and standard deviations are calculated.

3. Numerical simulations and results

The numerical simulations follow the rules of hard spheres simulations. First, the time t_i at which marbles $i-1$ and i collide is calculated as:

$$t_i = -\frac{\varepsilon_i}{v_i - v_{i-1}}. \tag{5}$$

These times are stored in a table T_t. The main part of the simulation is a loop over the collisions. One picks up, in the table T_t, the minimum time $t_{\min}$ which corresponds to the collision say $j-1$ and j. All the marbles evolve as free particles for the time $t_{\min}$, and at this time the collision between $j-1$ and j is treated via the matrix defined in equations (3) and (4). Then t_j is set to infinity, t_{j-1} and t_{j+1} are recalculated with the new velocities (5). The loop over the collisions is stopped when the minimum of the marble velocities is greater than the wall velocity. Figure 1 shows the trajectories of five identical marbles with the same initial separations ($\varepsilon_i = 1$) for various values of η. For each marble number n and inelasticity coefficient η, a thousand of random initial separations are used. In case II, masses and initial separations are chosen at random simultaneously.

The variations of $N_0(\eta)$ and $N_T(\eta)$ versus η are similar, both are monotonous increasing functions as η decreases. In the quasi-elastic regime ($\eta \simeq 1$), they are almost constant and equal to the elastic case I values: $N_0(\eta) = n$ and $N_T(\eta) = n(n+1)/2$. Both $N_0(\eta)$ and $N_T(\eta)$ diverge when η goes to a value η_2 which depends on n and very little on the mass distributions we used (the ratio $N_T/(nN_0)$ is always less than one). For $\eta < \eta_2$, the marbles stick to the wall, that means the velocities decrease exponentially with the collision number.

In the quasi-elastic regime, $\tau_b(\eta)$ is almost constant and equal to the elastic value $\tau_e = n\tau/2$, and the final distribution $\varepsilon_f(\eta)$ is the same as the initial one. All the marbles are ejected with close velocities. At smaller η, if other collisions are needed to complete the bounce, they arise at a much larger time ($\varepsilon_i/V_0 \ll -\varepsilon_i/(v_i - v_{i-1})$), and $\tau_b(\eta)$ and $\varepsilon_f(\eta)$ increase suddenly.

The variations of V_{CM} decrease monotonically as η decreases also. In case I, for $\eta = 1$, $V_{\mathrm{CM}} = V_0$ in the wall referencial, whereas its value goes to zero as η goes to η_2. In case II, the variations are essentially the same.

The most interesting quantities to look at are the relative kinetic energy and the velocity distribution function after the bounce. The relative kinetic energy is defined as:

$$E_k(n, \eta) = \sum_{i=1}^{n} \tfrac{1}{2} m_i (v_i - V_{\mathrm{CM}})^2. \tag{6}$$

In case I, for $\eta = 1$ and $\eta < \eta_2(n)$ this relative kinetic energy is zero. In the interval $[\eta_2, 1]$ this function reaches a maximum at $\eta = \eta_1$. In case II, the variations of $E_k(\eta)$ are the same, with the same coefficients η_1 and η_2 (see table 1), but $E_k(\eta)$ increases again as η approaches 1. We also checked the first moments of the velocity distribution function and found that in all cases they never approach those of a Maxwellian one, which in other words means that no thermalization occurs.

Table 1. Position η_1 and value $E_k^{\max}$ (in units of $m_0 V_0^2$) of the relative kinetic energy maximum versus the marble number n; η_2: value at which the marbles stay stuck to the wall; ICW: independent collision wave model; case I: identical marbles; case II: different mass marbles (with $\delta = 0.05$). Data for both cases I and II are from numerical simulations.

	η_1			$10^3 E_k^{\max}/n$		η_2			$3E_k/n\alpha\delta^2$
n	ICW	case I	case II	case I	case II	ICW	case I	case II	case II
2	0.4142	0.415	0.446	7.3	6.2	0.1716	0.172	0.177	1.01
3	0.5874	0.586	0.603	7.0	6.2	0.3333	0.339	0.339	1.09
4	0.6818	0.685	0.696	6.7	6.2	0.4465	0.454	0.455	1.03
5	0.7411	0.742	0.744	6.7	6.2	0.5279	0.534	0.536	1.12
6	0.7818	0.782	0.783	6.6	6.3	0.5888	0.594	0.595	1.08
7	0.8114	0.814	0.812	6.4	6.2	0.6360	0.639	0.640	1.32
10	0.8661	0.867	0.866	6.4	6.3	0.7295	0.725	0.725	1.17
15	0.9097	0.910	0.909	6.4	6.3	0.8107	0.800	0.801	1.35
20	0.9319	0.932	0.931	6.3	6.3	0.8545	0.857	0.845	1.44
30	0.9543	0.952	0.954	6.4	6.2	0.9005	0.902	0.897	1.80
50	0.9725	0.971	0.972	6.5	6.1	0.9391	0.944	0.943	2.70
100	0.9861	0.985	0.987	6.6	5.6	0.9691	0.972	0.971	6.60

So the numerical simulations show two phenomenological inelasticity coefficients: (i) at η_2, N_0 and N_T diverge, the marbles stick to the wall and the total relative kinetic energy vanishes (for $\eta > \eta_2$ the marbles rebound); (ii) η_t gives the maximum of $E_k(\eta)$. The main conclusion of the numerical results is that the phenomenological coefficients η_1 and η_2 depend very little on the mass distribution. Then it is of interest to study the case I analytically. This is the object of next section.

4. The independent collision waves (ICW) model

Here, we propose to study a model which reproduces with a good approximation some of the properties of $E_k(\eta)$. When the wall shocks the first marble, then the first marble will shock the second one and so on: this will be called here a *collision wave.* These collision waves are easily observed in figure 1.

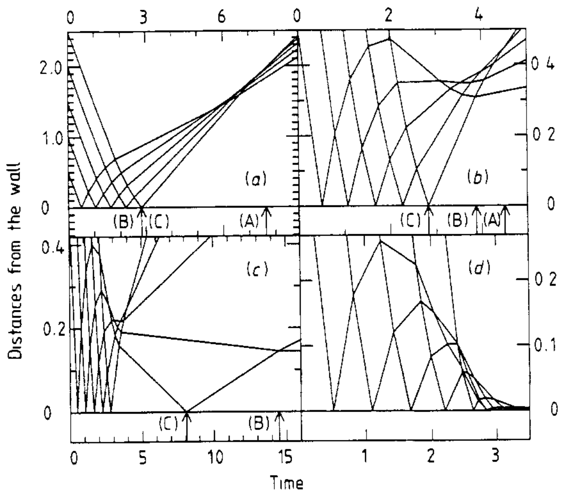

Figure 1. Set of five marbles trajectories corresponding to four values of the inelasticity parameter with same initial separations. The upper (a) (respectively (b)) quadrant is relative to $\eta = 0.8 > \eta_1$ (respectively $\eta_3 < \eta = 0.7 < \eta_1$) and the lower (c) (respectively (d)) one corresponds to $\eta_2 < \eta = 0.662 < \eta_3$ (respectively $\eta = 0.55 \simeq \eta_2$). Time is in units of τ. (A), (B) and (C) refer to different final state times defined in section 4.

Let X_n be the transfer matrix that gives the velocities after the pass of the collision wave in terms of the velocities before the collision wave. This matrix is an ordered product of binary collision $(n+1)\times(n+1)$ matrices $C_{i,i+1}$ defined as in equation (3) for lines and columns i, $i+1$ and identity matrix elsewhere:

$$X_n = C_{n-1,n}C_{n-2,n-1}\dots C_{1,2}C_{0,1}. \tag{7}$$

For the case of two marbles the collisions are always ordered: after a collision between the wall and 1, the next collision is between 1 and 2, collision followed by another one which is between the wall and 1, and so on until the two marble velocities are larger than V_0. Then, for two marbles the velocity distribution does not depend on the initial separations ε_i. For $n > 2$, this is not true and the effect of changing the initial separations is to change the collision order [6]; but the collision order will not change the final velocity distribution, except when two consecutive matrices are permuted because they do not commute in this case. However, it is possible to rearrange the matrix order to exhibit the X_n collision wave matrix, except when two waves interact as it is the case when a wave catches up a previous one. In the numerical simulation, at a given time, several waves propagate together and sometimes two waves interact: for example, in figure 1(c) we see that the second wave catches up the first one, whereas the waves propagate independently in figure 1(a), (b).

The independent collision waves (ICW) model neglects these wave interactions and allows us, as we shall see in the following, to give mathematical definitions to the 'critical' coefficients η_α which agree very well with the phenomenological ones.

4.1. Evaluation of $\eta_1(n)$ for identical masses

We have defined η_1 by the position of the $E_k(\eta)$ maximum, but this definition leads to a very complicated calculation; then, we use another very simple one: η_1 is the solution of $v_n(\eta)=v_w$ after the first wave. This equation is equivalent to $X_n[n+1, 1]=2\nu^n=1$, where $\nu=(1+\eta)/2$, and gives:

$$\eta_1(n)=2^{1-1/n}-1. \tag{8}$$

In order to understand why this value is close to $E_k^{\max}$ position, let us remark that two competitive effects are present: (i) when η decreases from 1, the velocities are different from each other after the bounce, so $E_k(\eta)$ increases; (ii) when the collision number increases, $E_k(\eta)$ decreases since after every collision the relative velocity decreases by the η factor (equation 2). Then a qualitative difference appears when the last marble is not ejected (i.e. $v_n<v_w$) after the first wave: see figure 1(a), (b). Table 1 provides a comparison of η_1 between this model and the simulation data. Note that this model is also in agreement with the results of case II. For large n, we have $\eta_1(n)\sim 1-2\ln(2)/n$.

4.2. Evaluation of $\eta_2(n)$ for identical masses

The calculation of η_2 is straightforward for two identical marbles. One calculates the eigenvalues of X_2 and finds that for $\eta>\eta_2=3-2\sqrt{2}\sim 0.1716$ there are two complex conjugate eigenvalues of modulus η and for $\eta<\eta_2$ there are two positive real eigenvalues $\lambda_-\in[0, \eta]$ and $\lambda_+=\eta^2/\lambda_-\in[\eta, 1]$. X_2 can be decomposed as:

$$X_2=[\mathcal{R}_{-\pi/4}\mathcal{A}_{0x}(\eta)]^2 \qquad \text{where} \qquad \mathcal{A}_{0x}(\eta)=\begin{pmatrix}\eta & 0\\ 0 & 1\end{pmatrix} \tag{9}$$

X_2 acts on the two-dimensional marble velocity space and $\mathcal{R}$ is a rotation. If the initial condition is the point $A_0=(-V_0, -V_0)$ and the wall velocity is zero, the successive actions of X_2 give a sequence of points A_j. When one point A_j reaches the upper right quadrant, no more collisions occur between the wall and the marbles. $\mathcal{A}_{0x}$ has an effective action of rotating OA_0 by a positive angle ($\eta<1$) which competes with the pure reaction $\mathcal{R}_{-\pi/4}$. When the eigenvalues are complex, the pure rotation wins and the sequence reaches the upper right quadrant. When real eigenvalues appear, the vector OA_0 can be projected on the two eigenvectors which are in the first upper right quadrant and since the two eigenvalues are less than 1, the sequence converges towards the origin, i.e., the marbles stick to the wall.

For larger number of marbles, these ideas remain the same, we look for the value of η at which a real eigenvalue appears in X_n. First, one can show the following recurrence relation for $x_n=\det(X_n-\lambda I)$, where I is the identity matrix:

$$x_n+(\lambda+\eta)x_{n-1}+\lambda\nu^2x_{n-2}=0 \tag{10}$$

where $\nu=(1+\eta)/2$. With the conditions $x_1=-(\eta+\lambda)$ and $x_2=(\eta+\lambda)^2-2\lambda\nu^2$, the determinant can be written as: $x_n=r_-^n+r_+^n$, where $r_\pm=[-(\lambda+\eta)\pm\sqrt{(\eta+\lambda)^2-4\lambda\nu^2}]/2$

are the solutions of equation (10), the form of which is $x_n = r^n$. The zeros of x_n are then given by the solutions of $(r_+/r_-)^n = -1$:

$$\frac{r_+}{r_-} = z_p = \exp\left(\frac{i\pi(2p+1)}{n}\right) \qquad 0 < 2p+1 \leq n. \tag{11}$$

Then replacing $r_\pm$ by their expression in equation (10), we obtain a set of second order polynomials in λ:

$$\lambda_p^2 - 2\lambda_p\left[(1-\nu)^2 + \nu^2 \cos\frac{\pi(2p+1)}{n}\right] + \eta^2 = 0. \tag{12}$$

The eigenvalues λ_p are real if $\eta \in [0, \eta_p]$ where

$$\eta_p = \tan^2\left(\frac{\pi}{4}\left(1 - \frac{2p+1}{n}\right)\right).$$

The first real eigenvalue to appear is:

$$\eta_2 = \tan^2\left(\frac{\pi}{4}\left(1 - \frac{1}{n}\right)\right) \tag{13}$$

One can show that when the eigenvalues are real, we have $\lambda_{p-} \in [0, \eta]$ and $\lambda_{p+} \in [\eta, 1]$, whereas $|\lambda_p| = \eta$ when they are complex. These properties imply that the marbles stick to the wall when $\eta < \eta_2$.

Table 1 provides a comparison of η_2 given by this model and by the simulation data. Note that the results of this model are also in agreement with those of the case II. For large n, we have $\eta_2(n) \sim 1 - \pi/n$.

In the case where the wall is replaced by a marble, the same treatment holds and the 'critical' value η_2 becomes: $\eta_2'(n) = \tan^2(\frac{1}{4}\pi(1 - 2/n))$. Note that, for large n, the values taken by η_2' are very close to those given by η_2 but for $n/2$, i.e.: $\eta_2'(n) \simeq \eta_2(n/2)$.

4.3. *Evaluation of $\eta_3(n)$ for identical masses*

The 'critical' value $\eta_3(n)$ is obtained when the number of waves N_0 increases by 1 from its elastic value n. This happens when $v_1(\eta) = v_w$ after n first waves (see figure 1(*b*), (*c*)). The equation to solve is $X_n^n[2, 1] = 1$. We found no evident analytical solution for this algebraic equation. Knowing that $\eta_2(n) < \eta_3(n) < \eta_1(n)$, we have obtained an empirical fit, the form of which is similar to that of $\eta_2(n)$: $\eta_3(n) \simeq \tan(\frac{1}{4}\pi(1 - a/n))$ where a is around 1.16.

4.4. *The relative kinetic energy in case I*

It is possible to choose different final states for this study, depending on the stop test of the simulation. One can consider a first state (A) when no more collisions can occur in the marble aggregate (corresponding to t_∞), another one is the state (C) when no more collisions with the wall occur. The present study concerns the state (B) ($v_i > v_w$ for all i), state which is intermediate to the two previous ones. In figure 1 are shown the times corresponding to these different stop tests. For each state is associated a different relative kinetic energy curve E_k. The variations of $E_k(\eta, 30)$ for the three states are plotted in figure 2.

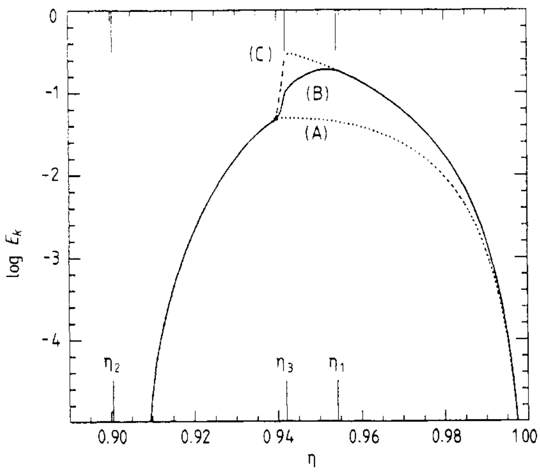

Figure 2. Comparison of the log $E_k(\eta)$ curves for $n=30$, (E_k in units of $m_0V_0^2$), in case I and for the three final states: state (A) (dotted line), state (B) (full line), state (C) (broken line) defined in section 4.

Case (A) is well reproduced by the ICW model. One propagates the waves one after the other as long as it is possible, i.e., as long as the relative velocity of two consecutive marbles is negative. In this model, two consecutive colliding waves are replaced by only one wave and the initial separation distribution plays no role. Note that for $\eta_3<\eta<1$, this curve is also reproduced by applying the $(X_n)^n$ matrix, i.e., exactly n complete waves.

Cases (B) and (C) are identical in the quasi-elastic regime $(\eta_1<\eta<1)$. This can be understood by analysing the collision timescales. In this regime the ejected marbles have almost the same velocities (see figure 1(a)), which implies that their relative velocities are much smaller than their own velocities. The bounce duration τ_b is of the order of $\tau_e=n\tau/2$, whereas the time $t_i=-\varepsilon_i/(v_i-v_{i-1})$, at which a future collision between ejected marbles will occur, is much larger and consequently the stop test is the same for (B) and (C) as long as $\tau_b<t_i$. In terms of the ICW model, this is expressed by an n-wave matrix: the first wave X_n ejects the nth marble, the second one X_{n-1} ejects the $(n-1)$th and so on. The matrix which gives the final state, at $\eta=1$, is $T=X_1X_2\ldots X_n$. Curve (C) is reproduced by this matrix up to $\eta\simeq\eta_3$ where E_k reaches its maximum. (A), (B) and (C) fall on the same curve for η just less than η_3. The only parts of (B) and (C) curves which are not reproduced by the ICW model are those for η lying between η_3 and η_1.

4.5. *The elastic relative kinetic energy in case II*

It is remarkable that the variations of E_k are very close for both cases I and II, except for η close to 1. In particular, the respective values of η_1 and η_2, for each case, differ only by few per thousand (see table 1). However, when η approaches 1 the $E_k(\eta)$ curves change completely. Indeed, in case II at $\eta=1$ the velocities are not the same after the bounce so E_k is different from zero. Let us recall that the masses are given by $m_i=m_0(1+\delta\xi_i)$ where ξ_i is a random number in $[-1, 1]$. At $\eta=1$, E_k is not sensitive to the initial separation distribution as long as the mass dispersion parameter δ is not too large. Then E_k is only a function of n and δ.

The final states (B) or (C) are given by applying the n-wave T matrix, whereas state (A) is given by $(X_n)^n$; when applied to the initial condition where the plan velocity is V_0 and $v_i(t=0)=0$, the final velocities are

$$v_i = 2V_0(1-0.5\varepsilon\delta(\xi_i - \xi_{n-i})) + \mathcal{O}(\delta^2)$$

where $\varepsilon=+1$ (respectively -1) for (B) and (C) (respectively (A)). This implies that $V_{\mathrm{CM}} = 2V_0(1+\mathcal{O}(\delta^2))$ and E_k reads $E_k(n,\delta)=\frac{1}{3}m_0V_0^2n\delta^2\alpha+\mathcal{O}(\delta^4)$ where $\alpha=1$ if n is even and $(1-1/n)$ if n is odd. In table 1, one can see that the deviations from this first term arise for $n\delta>1$.

5. Conclusion

We have shown that the simple model of independent collision waves is able to exhibit the main properties of a marble column bounce in the absence of external field. The quasi-elastic regime as well as the damped regime are reproduced by a simple ICW matrix formalism. This leads to computational times of orders of magnitude smaller than those given by a direct simulation, since no statistical average is needed to be done. The results can be summarized as follows: η_2 is the value where the collision wave number diverges, η_1 is solution of $v_n = v_w$ after the first collision wave, whereas η_3 is solution of $v_1 = v_w$ after the nth wave.

For large n, the predicted values of η_1 and η_2 vary similarly, and the maximum of the relative kinetic energy $E_k(\eta, n)$ is almost proportional to the marble number n. This suggests that $E_k(\eta, n)$ can be deduced from a universal curve $\mathcal{F}$ using the following renormalization:

$$E_k(\eta, n) = n\mathcal{F}(x) \qquad \text{where} \qquad x = n(\eta - \eta_1(n)). \tag{14}$$

The functions $\mathcal{F}(x)$, obtained from the numerical data, are plotted in figure 3 for $n = 10$, 50 and 100, for both cases I and II.

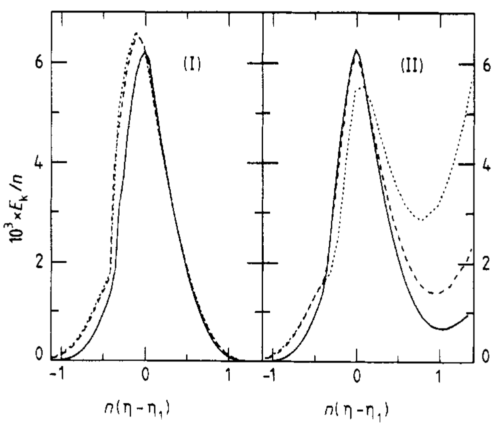

Figure 3. Comparison of the reduced curves $\mathcal{F}(x) = E_k(\eta, n)/n$ as functions of $x = n[\eta - \eta_1(n)]$. The left (respectively right) curves group corresponds to case I (respectively case II) for $n = 10$ (full line), $n = 50$ (broken line) and $n = 100$ (dotted line).

In case I and for large n, these curves converge towards a single one. This implies that changing η is equivalent to change n: the relative kinetic energy per particle is the same as long as $n(\eta - \eta_1(n))$ is the same, which reads $(1-\eta)n$ = constant for large n. Consequently, the properties of a one-dimensional marble column depend on the only parameter $\gamma = \mathcal{I}n$, where $\mathcal{I} = 1 - \eta$. Doubling the number of marbles has the same effect as having $\mathcal{I}$ divided by 2. This conclusion remains the same when the wall is replaced by a marble. In case II, the functions $\mathcal{F}(x)$ are the same as in case I in the region $x < 0 (\eta_2 < \eta < \eta_1)$, but are different in the quasi-elastic regime.

We are presently investigating the effect of gravity on the dynamics of the same column that is free to bounce on a vibrating plan; a similar work, concerning one bouncing marble at $\eta = 0$, has been studied recently [9]. Except for the collision between the wall and the first marble, the properties of the collision waves are still valid.

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